Correcting for bounded bandwidth when estimating tissue attenuation from mean frequency downshift

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Abstract - **The attenuation of tissue can be estimated utilizing the downshift of the center frequency of a propagating pulse. In general it is assumed that the shape of the emitted pulse can be approximated by a Gaussian function and attenuation is assumed to change linearly with frequency. At this conditions the downshift of the mean frequency of pulse spectrum depends linearly on attenuation coefficient, pulse bandwidth and propagation distance. This is a good approximation for relatively narrowband pulses and small penetration depth. But for short pulses and deep penetration the frequency downshift is large and the ultrasonic pulse is no more Gaussian, thus the previous assumption is no longer correct. The closer is the mean frequency of the pulse to the lower frequency bound of the receiving system the bigger deformation of the pulse spectrum occurs and consequently the attenuation is determined with bigger error. The following paper presents how to correct the experimentally determined mean frequency and to obtain reliable results when investigating tissue attenuation with wideband pulses. We propose a new formula for the dependence between pulse mean frequency, tissue attenuation, pulse bandwidth and traveled distance. The formula was derived from the mean frequency of Gaussian pulse spectrum determined in the limited frequency band. The formula was applied to simulate variation of mean frequency MF of the pulse propagating in the medium with attenuation coefficient corresponding to the attenuation in the tissue mimicking phantom. The MF was also determined (using the correlation estimator of MF and next trend extraction using Single Spectrum Analysis) from the simulated ultrasonic echoes and echoes scattered in the tissue phantom. The corrected non-linear formula describes well MF variation along the pulse propagation path. The departure from the linear dependence increases with large MF shift, thus it is well pronounced for highly attenuating tissue, the wideband pulses and deep penetration.**

I. INTRODUCTION

The visualization of the attenuation coefficient of a tissue could potentially add a new quality to the US imaging. It has been demonstrated that pathological tissue has different attenuation properties than the healthy one. Oosterveld et. al. have shown that the slope of attenuation coefficient, combined with statistical parameters of image texture can be used to diagnose the diffuse liver disease [1]. Saijo et. al. employed scanning acoustic microscope to measure five types of gastric cancer and indicated different attenuation coefficient and sound speed comparing to normal tissue [2]. Bigelow et. al. investigated possibility of the prediction of the premature delivery based on the noninvasive ultrasonic attenuation determination in rats [3]. McFarlin et.al carried out the similar investigation for humans [4]. In various other publications it has been reported that pathological processes can lead to changes in the mean attenuation coefficient that range from several percent for cirrhotic human liver, through dozens percent for fatty human liver [5], or degenerated bovine articular cartilage [6] to over a hundred percent in case of *in vivo* porcine liver after a HIFU treatment [7] up to two hundred percent for porcine kidney thermal coagulation [8]. These reports prompt to develop the attenuation estimation and visualization procedure, which could be implemented in ultrasound system working in pulse echo mode.

In our approach the mean frequency MF is directly evaluated from the backscatter. The common assumption of the Gaussian pulse spectrum leads to linear relation between the attenuation and MF shift. However, this relation does not takes into account the limited bandwidth of the real ultrasound system, which leads to the non-linear relation between attenuation and the frequency shift. This paper presents the non-linear formula for MF resulting from assumed Gaussian pulse spectrum and limited frequency bandwidth of the system.

II. METHODS

The ultrasonic wave propagating through the soft tissue is attenuated due to absorption and scattering. The amplitude A of the wave exponentially decrease with the propagation distance, what can be expressed as

$$A = A_0 \exp(-\alpha(f) \cdot x) \quad (1)$$

where A_0 – initial amplitude, $\alpha(f)$ – frequency dependent attenuation coefficient and x – the distance passed by the wave within the tissue. In the soft tissue the attenuation coefficient depends on frequency and can be described by the following empirical expression [9]

$$\alpha(f) = \alpha_0 f^n \quad (2)$$

where attenuation α_0 corresponds to the frequency equal 1MHz and n is a positive exponent. The value of n is typically close to 1 for soft tissues and it is common to assume that $n=1$, i.e. that attenuation increases linearly with frequency [9]. According to (2) the higher frequency components of a propagating pulse are attenuated stronger than lower frequency components. This results in the shift of the pulse mean frequency f_m towards the lower frequencies. Assuming the Gaussian pulse spectrum, the f_m can be expressed as follows [10]

$$f_m = f_0 - \frac{\alpha_0 \Delta x \sigma^2}{2} \quad (3)$$

where f_0 is the initial pulse mean frequency, σ^2 is the Gaussian variance of the pulse spectrum corresponding to spectrum width, Δx denotes penetrated distance and α_0 is the attenuation coefficient. Gaussian pulse spectrum preserves the shape during propagation in linearly attenuating medium i.e. the σ^2 is constant. The attenuation coefficient can be calculated from (3) as follows

$$\alpha_0 = -\frac{2}{\sigma^2}\frac{\Delta f_m}{\Delta x} \xrightarrow[\Delta x \to 0]{} -\frac{2}{\sigma^2}\frac{d f_m}{dx} \quad (4)$$

The equation (3) is valid only for pulses that amplitude spectra can be approximated with Gauss function. In practice, most of the spectra of scanning pulses are close to the Gaussian shape. For the Gaussian pulses (pulses with Gauss-shaped spectra) the mean frequency f_m of the spectrum is identical with the peak frequency – f_p (frequency component of maximum amplitude). Usually, the frequency-band of the receiving systems overlaps most of the transmitted pulse frequency-band. For low attenuating tissue and short propagation distance the spectrum of the received echoes is similar to the transmitted pulse spectrum. For high attenuation and deep penetration, most of the pulse energy is located in the low frequency components of the pulse spectrum, some of them being out of the receiving system bandwidth. Thus, even though the ultrasonic pulses still are Gaussian, the received pulses are deformed and their spectra differ from the Gaussian shape. The measured mean frequency differs from the mean/peak frequency of the propagating ultrasonic pulse thus resulting in attenuation error when the measured data are put into (4). We propose a modified expression that allows to restore the value of the mean frequency of ultrasonic pulse from the measured data. The basic idea is graphically explained in Fig. 1. When the pulse spectrum is close to the bandwidth border the mean frequency differs significantly from the peak frequency. The more accurate expression for the mean frequency can be developed as follows. Let's assume that the Gaussian pulse is filtered by the weighting window with cut off frequencies f_1 and f_2. The power spectrum of the propagating pulse is expressed by (5)

$$S(f) = A \exp\left(-\frac{(f-f_m)^2}{\sigma^2}\right) \quad (5)$$

where A is a constant. The mean frequency of the pulse spectrum windowed by the receiving system is given by (6).

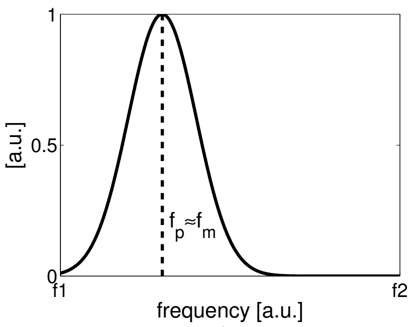
$$f_{m1} = \frac{\int_{f_1}^{f_2} f S(f) df}{\int_{f_1}^{f_2} S(f) df} \quad (6)$$

The integration of the above expression leads to the new formula for the mean frequency f_{m1}. The new mean frequency expression is given by

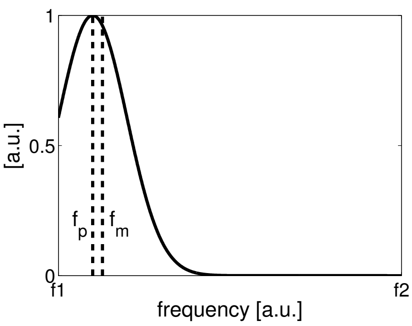
$$f_{m1} = f_m - \frac{\sigma}{\sqrt{\pi}} \frac{\exp\left(-\frac{(f_2-f_m)^2}{\sigma^2}\right) - \exp\left(-\frac{(f_1-f_m)^2}{\sigma^2}\right)}{\mathrm{erf}\left(\frac{f_2-f_m}{\sigma}\right) - \mathrm{erf}\left(\frac{f_1-f_m}{\sigma}\right)} \quad (7)$$

Where function $erf()$ is defined by (8) and f_m is given by (3).

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2)\, dt \quad (8)$$



a)



b)

Fig. 1. The received pulse spectrum windowed by the receiving system for low attenuating tissue and short propagation (a) and for high attenuating medium and long propagation (b).

Hence, the new expression describing the mean frequency f_{m1} as the sum of the "old" mean frequency (given by (3)) and the correction component. The verification of new mean frequency expression is presented in the next section.

In our approach the measured mean frequency MF was estimated from backscattered echoes using the correlation estimator (IQ algorithm). The estimator is depicted by

$$MF = \frac{1}{2\pi T_S}\tan^{-1}\left(\frac{\sum_{i=1}^{N}(Q(i)I(i+1)-Q(i+1)I(i))}{\sum_{i=1}^{N}(I(i)I(i+1)+Q(i+1)Q(i))}\right) \qquad (9)$$

where T_s is the sampling period and N is the estimator window length. The Q and I are quadrature and in-phase signal components and are obtained by quadrature sampling technique. The signal samples are numbered by index i. The quadrature sampling is often used in modern scanners and the correlation estimator is widely used in Doppler techniques [11]. The MF line is created calculating the mean frequency from the raw backscattered echo data. The MF lines are characterized by high variance due to random character of signal backscattered in soft tissue. Therefore, the direct use of (4) results in the highly noised, not acceptable attenuation estimates. The reduction of the MF line random variability is realized by the moving average filtration and the Singular Spectrum Analysis (SSA) technique. The moving average filter operates over adjacent MF lines in lateral direction. The filter window is related to the lateral resolution of the estimate. Then the SSA trend extraction algorithm operates along the MF lines in axial direction. The aim of the SSA technique is the decomposition of the input data series into the sum of components which can be interpret as the trend, oscillatory components and the noise (non-oscillatory components). Details of this technique can be found in the literature [12]. The final attenuation estimates are enumerated from the MF lines using (4), assuming model (3) or (7).

III. RESULTS

The proposed new estimate of mean frequency was verified using simulated and experimental RF data. The backscattered RF line were simulated as follows: the uniform random scatterer distribution over the depth of 20cm was generated. The scattering function was simulated by the Dirac's delta functions with delay corresponding to the position of the scatterers. The amplitudes of the scatterers were given by uniform random distribution. The backscatter was simulated as the convolution of the impulse response of the transducer with the scattering function. The 30 backscatter RF lines were simulated for two separate cases – the transmitted pulse bandwidth equal 75 and 100% assuming the medium attenuation equal 0.5 and 1 dB/(MHz·cm), respectively. Next, the MF trends were enumerated from simulated backscattered echoes using procedure depicted in the *Methods* section. The 30 MF lines were averaged. Additionally, the f_m and f_{m1} changes predicted by (3) ("old" model) and (7) ("new" model) were calculated. The results are presented in Figs. 2 and 3. The difference between these two models of the frequency shift is well pronounced for the broad bandwidth pulses and highly attenuating materials (Fig. 3). Both equations give very similar results for narrow bandwidth pulses and low medium attenuation (Fig. 2) – here the slight difference becomes to be visible only for high penetration depths exceeding 15cm.

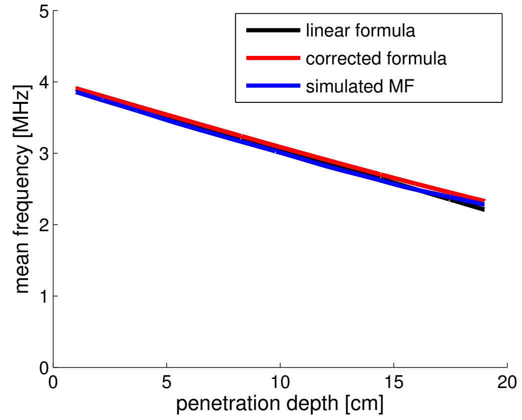


Fig. 2. Mean frequency versus penetration depths simulated for pulse bandwidth 75% and attenuation coefficient 0.5 dB/(MHz·cm); black, red and blue lines are the mean frequencies predicted by (3) (old model - f_m), (7) (new model – f_{m1}) and estimated from simulated echo data (MF), respectively.

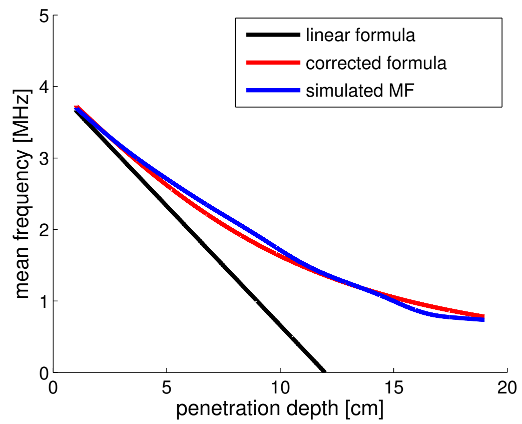


Fig. 3. Mean frequency versus penetration depths simulated for pulse bandwidth 100% and attenuation coefficient 1 dB/(MHz·cm); black, red and blue lines are the mean frequencies predicted by (3) (old model - f_m), (7) (new model – f_{m1}) and estimated from simulated echo data (MF), respectively .

The measurements were made using flat transducer (IMASONIC ME 1/2'') and the uniform tissue phantom with attenuation coefficient equal 0.7 dB/(MHz·cm). The initial mean frequency of the transmitted pulse was 5.6 MHz and the pulse bandwidth equal 100%. The 33 backscattered RF lines from uniform tissue phantom were collected and average mean frequency trend was calculated. Theoretical mean frequency changes predicted by (3) and (7) were calculated as well. The results are presented in Fig. 4.

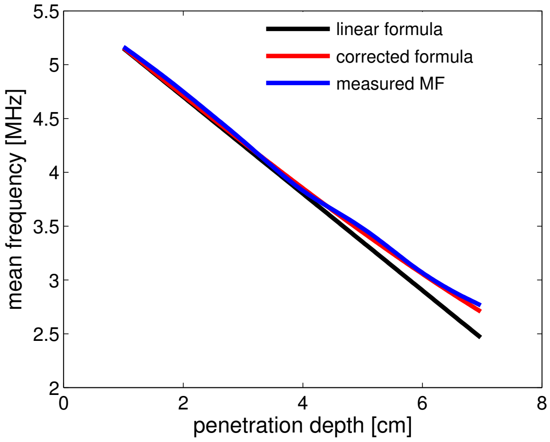


Fig. 4. Mean frequency versus penetration depths measured for pulse bandwidth 100% and attenuation coefficient 0.7 dB/(MHz·cm); black, red and blue lines are the mean frequencies predicted by (3) (old model - f_m), (7) (new model – f_{m1}) and estimated from measured echo data (MF), respectively.

IV. Conclusions

The corrected non-linear formula describes well mean frequency variation along the pulse propagation path and it can be used for the attenuation coefficient estimation. The departure from the linear dependence is well pronounced for highly attenuating tissue, the wideband transmission and increases with penetration depth. It was shown that corrected formula fits better both simulated and measured data but for narrowband pulses, short propagation distances and medium with low attenuation the differences are negligible. However, the corrected formula is more complex. Therefore, for such propagation parameters the linear formula is more convenient. As the deviation from the linear formula is evident for the wide band pulses, deep penetrations and highly attenuating materials, for such parameters the corrected formula should be considered. For example, a ~3% error of mean frequency (attenuation coefficient error ~13%) may be expected when non-corrected formula is applied to the mean frequency shift estimated from the pulse (transmitted frequency equal 5.6 MHz, FWHM equal 70%) scattered at the 5 cm depth of the tissue phantom with attenuation of 0.7 dB/(MHz·cm).

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